Study of the enhanced algorithm for control information dissemination in Wi-Fi Mesh networks

Conference Paper · September 2016

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***Abstract*—Various networking protocols currently used in wireless networks disseminate much control information. In particular, the deterministic channel access protocol (called MCCA) specified in the IEEE 802.11 standard sends information about channel reservations. The standard proposes a novel approach to reduce this overhead, which is based on dividing various pieces of control information into several groups. When the content of some group changes, the station sends a differential update related to this group. However, a group management algorithm (GMA)—which directly affects the performance of such an approach—is not specified in the standard. Previous studies propose a simple GMA which is based on the idea to use as low number of groups as possible. However, when the content of groups changes very fast, the overhead is high. In this paper, we propose and study an enhanced GMA which manyfold reduces the overhead. We develop an analytical model to evaluate the proposed GMA performance and to find its optimal parameters. We also obtain an explicit formula for the optimal parameters in the asymptotic case. We show that such asymptotically optimal parameters, which do not depend on traffic parameters, provide low overhead in non-asymptotic case as well. Thus, the proposed GMA can be easily implemented without any complex method for parameters adjustment.**[1]

I. Introduction

Many protocols currently used in wireless networks generate and disseminate much control information, consuming a significant part of channel resources [1]. Instead, these resources could be utilized for user data transmissions. Therefore, an important task is to reduce such an overhead.

The simplest approach to disseminate control information (called *full dump*) is to broadcast messages containing all the information even if it is not changed. Such a redundancy makes the full dump approach resistant to packet losses but leads to high overhead. This approach is used in the proactive routing protocol OLSR [2], where each station (STA) periodically broadcasts information about all advertised links [3]. The distributed MAC protocol for ad hoc networks based on TDMA RR-ALOHA [4] uses a similar approach: every packet contains the information about busy and free time slots.

Another popular approach is to send incremental messages (also called differential updates) containing only changes of the information since the last full dump message. Sending small incremental messages instead of some full dumps significantly reduces overhead. For example, such an approach is used in various routing protocols, e.g. DSDV [5], TBRPF [6], OSPF-MDR [7] and PSR [8]. To refer to a particular piece of information in an incremental message, STAs label various pieces of information with unique identifiers (IDs). However, when the total number of pieces of information is large, the length of ID becomes very high. Eventually, in a dynamic network with very frequent topology and traffic changes, this significantly increases the amount of sent control information.

One of the ways to address this problem is to join various pieces of information into a relatively small number of groups. Then the STA sends information only about the changed groups and includes the ID of the group (instead of the IDs of each piece of information) in incremental messages. In particular, such an approach is employed in the IEEE 802.11 standard [9]. The standard introduces a novel channel access protocol called Mesh coordination function Controlled Channel Access (MCCA), which allows STAs of a wireless multihop network (Wi-Fi Mesh network) to reserve periodic time intervals for data transmission [10]. A sequence of periodic time intervals is called reservation. During reserved time intervals, only two STAs can transmit data and acknowledgements while their neighbors cannot transmit. To avoid reservations overlapping, each STA shall broadcast control information about reservations already established by this STA and its neighbors. This procedure is called reservation advertisement and it is described in details below.

According to the standard, all the reservations which STA shall advertise are divided into several groups. Here group is a certain subset of reservations (including empty subset). Each reservation belongs to only one group. The total number of groups is limited by parameter G (by default, $G = 16$). Each STA periodically broadcasts a special management frame called beacon. In each beacon, a STA includes a short message called Advertisement Overview to notify neighboring STAs about non-empty groups. This message contains the following fields: (i) G-bit bitmap, in which bit i equals 0 if group i is empty, and equals 1 otherwise; (ii) Sequence Number (SN), which identifies the version of the bitmap.

When an empty group is filled (some reservations are put into this group), the corresponding bit in the bitmap is changed from 0 to 1, and the STA sends information about the content of this group by including it in a beacon. If at least one reservation is deleted from a group, the whole group shall be cleared, the corresponding bit shall be changed from 1 to 0, and the remaining reservations shall be moved to one or several empty groups. If a STA needs to fill a group, but no bits in the

[1]The research was done in IITP RAS and supported by the Russian Science Foundation (agreement No 14-50-00150).

bitmap can be changed from 0 to 1, the SN is increased, all groups become empty, and all the reservations are divided into groups from scratch. In this case, the STA sends information about all the reservations, since all the groups are changed.

With the same SN, the content of the group cannot be modified, i.e. the STA can fill the group and clear it only once. So, with the same SN, each bit can be changed from 0 to 1 and then from 1 to 0 only once. This restriction is introduced to avoid the ambiguity problem, which could be caused by an unsuccessful transmission of the Advertisement Overview message (for example, because of noise in the channel). Let us consider an example. STA A has not received successfully an Advertisement Overview message including information about some cleared group. So, the STA A is waiting for the next Advertisement Overview message. If the neighboring STA B will fill this group with reservations without changing SN, the STA A will not be able to determine whether this group is changed or not, because for STA A the corresponding bit will not change.

Neighboring STAs process received messages describing reservations as follows. A STA compares the SN value and the bitmap with the previously received ones. If only the bitmap is changed, the STA (i) removes information about reservations of the groups for which the corresponding bits have been changed from 1 to 0, and (ii) adds information about reservations of the groups for which the corresponding bits have been changed from 0 to 1. Otherwise, if the SN is changed, the STA (i) removes information about all the reservations, previously received from the neighbor, and (ii) adds information about reservations of all non-empty groups indicated in the bitmap since the content of all groups is changed. The STA can request the information about a certain non-empty group from the neighbor by sending a special management frame if this information has not been received.

Note that with the described above group-based approach, the amount of advertised information (the number of reservations put into beacons) significantly depends on how STAs group reservations. However, the standard does not specify any group management algorithm (GMA). In [11] the authors have proposed a simple GMA which is based on the idea to use as low number of groups as possible. The results show that when the content of groups changes very fast, the proposed GMA leads to huge overhead which is comparable with that of the full dump approach. In this paper, we propose and study an enhanced GMA, which tries to maintain a predefined number K of groups containing reservations. When K is low, the probability of closing at least one reservation from every group is high, which leads to frequent full dumps. On the contrary, when K is high, STA frequently changes SN and as a result makes full dumps. In Section V, we will show that the proper choice of K (and also the distribution of reservations into these K groups) allows to significantly reduce the amount of advertised information. For that, we develop an analytical method for estimating the values of the GMA parameters which provide the minimal amount of advertised information. We also investigate the behaviour of the proposed GMA in the asymptotic case and explicitly find the optimal parameter K. We show that the proposed enhanced GMA significantly outperforms the GMA proposed in [11].

The rest of the paper is organized as follows. In Section II, we specify the proposed GMA. We carry out a preliminary analysis of the proposed GMA in Section III. In Section IV, we develop an analytical method to find the optimal values of the GMA parameters. In Section V, we evaluate the performance of the enhanced GMA and compare it with the GMA proposed in [11]. Section VI concludes the paper.

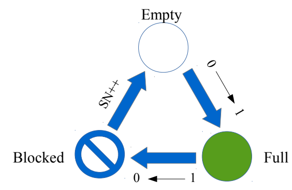

Fig. 1. Group life cycle.

II. Group Management Algorithm

To describe the proposed GMA, let us introduce some notations. With a given SN, each group can be in one of the following three states:

- *Empty (E)*. The group has not contained reservations under this SN yet. The corresponding bit in the bitmap equals 0.
- *Full (F)*. The group contains at least one reservation, and the corresponding bit in the bitmap equals 1.
- *Blocked (B)*. The group does not contain reservations (the corresponding bit in the bitmap equals 0), but under the same SN it previously contained reservations. Thus, no reservations can be put in this group until the SN changes.

We consider the group states at the beginnings of the beacon intervals. Fig. 1 shows how groups change their states during the operation of any GMA. Consider a group in state F. If one or several reservations in this group are closed, the group changes its state to B and all remaining reservations from this group are moved to one or several empty groups. In the bitmap, the bit corresponding to this group changes from 1 to 0. After that, the group may change its state only when the SN increases. When this occurs, the group (as well as other groups) changes its state to E, and thus, reservations can be put in it again. Note that the SN increases and reservations are rearranged into groups in the same beacon interval. Thus, for some groups the transitions from state B to state E and then from state E to state F can occur in the same beacon interval.

We introduce the following variables:

1) g_F is the number of groups in state F;
2) g_E is the number of groups in state E;
3) r is the current number of reservations which are advertised by STA;
4) n is the number of new reservations set up (at the STA and its neighbors) in the previous beacon interval;
5) d is the number of reservations closed in the previous beacon interval.

We propose a GMA, which tries to maintain K full groups. When $K = 1$, the algorithm corresponds to the one proposed in [11]. Let us describe in detail how the algorithm works at the beginnings of each beacon interval.

If $n = 0$ and $d = 0$, all the groups do not change. Hence the SN and the bitmap do not change. The STA does not send any new information about reservations, i.e., the amount of advertised information equals 0. Otherwise, the proposed GMA works as follows.

1) Groups in which at least one reservation is closed change their states to B. Let b be the number of such groups ($b \le g_F$). The new value of g_F equals $g'_F = g_F - b$.
2) Let r_b be the number of remaining reservations from the blocked groups. If $\tilde{r} = r_b + n = 0$, no more actions are done and the amount of advertised information equals 0.
3) Otherwise, if $\tilde{r} > 0$, reservations shall be arranged into empty groups in the following way.
   a) If $g_E > 0$, then several cases are possible.
      i) If $g_F \ge K$, then $\tilde{r}$ reservations are arranged into one empty group. So $g'_F = g_F + 1, g'_E = g_E - 1$.
      ii) If $g_F < K$, then $\tilde{r}$ reservations are arranged uniformly into $\tilde{g} = \min(K - g_F, g_E, \tilde{r})$ empty groups. After that, $g'_F = g_F + \tilde{g}, g'_E = g_E - \tilde{g}$.
   b) If $g_E = 0$ then the SN increases. All existing r reservations (including established in the previous beacon interval) are arranged into the first $\min(K, r)$ groups uniformly. After that, $g'_F = \min(K, r), g'_E = G - g'_F$.

Note that g_F does not always equal K. For example, when in the previous beacon interval $g_F = K$, $g_E > 0$, $d = 0$ and $n > 0$, in order to minimize the amount of advertised information the GMA arranges n reservations into one group and hence $g'_F = K + 1$. If $g_F = K$, $n = 0$ and all the reservations from one group are closed during the previous beacon interval, the GMA does not change other groups and the number of full groups g_F becomes equal to $g'_F = K - 1$.

III. Preliminary Analysis

To evaluate the performance of the proposed GMA, let us consider the following scenario. As mentioned in Section I, a STA advertises all the reservations established by the STA and its neighbors. STAs establish reservations when new data flows appear and close them when the corresponding data flows are finished. Let the total number of new data flows appearing at the considered STA and its neighbors in a beacon interval have the Poisson distribution with mean value λ. The lifetime of each data flow and the corresponding reservation has the exponential distribution with rate parameter μ. We assume that all new reservations are established at the end of the beacon interval and therefore their lifetime starts to count down at the beginning of the next beacon interval. Following the standard [9], we limit with threshold R the number of reservations that each STA can track and advertise. We also assume that beacons carrying information about reservations are delivered to all neighboring STAs reliably, so this information is not retransmitted.

In this paper, we estimate the amount of advertised information V as the average number of reservations information about which is put into the beacon. As the size of an Advertisement Overview message (SN and bitmap) is much less than the size of a message describing one reservation, we neglect regular transmissions of the Advertisement Overview messages.

Fig. 2 shows the results obtained by simulation for different K, μ, and λ values. In all experiments, we set $R = 100$ and $G = 16$. We can see that optimal K, i.e., the value that minimizes the amount of advertised information, depends on μ and λ. However, one can notice, that the optimal value of K found in the area where $\lambda \to \infty$ (which corresponds to $\rho = \lambda/\mu \to \infty$) provides low amount of advertised information for other values of λ. We will also confirm this statement in Section V. Thus, it is worth to consider in detail the area where $\lambda \to \infty$. Let us say that in this area the STA works *in saturation*.

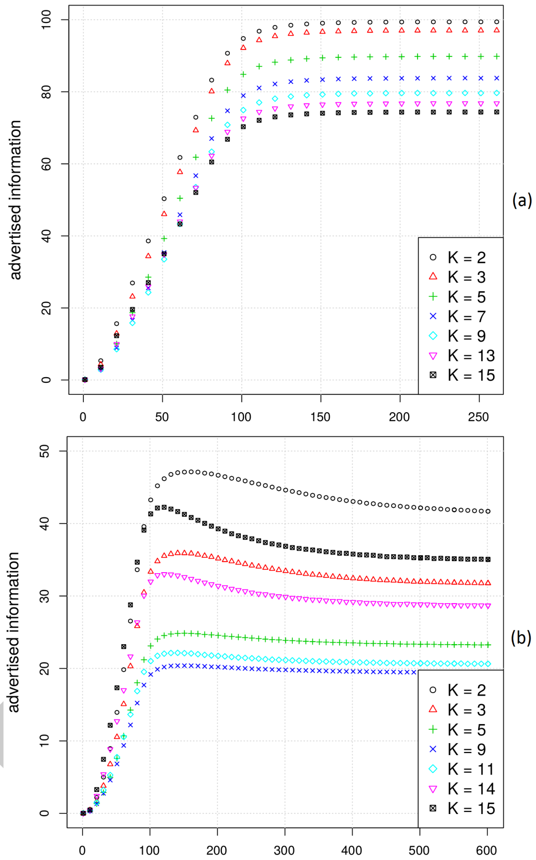

Fig. 2. The mean amount of advertised information (obtained via simulation) vs. $\rho = \frac{\lambda}{\mu}$ when (a) $\mu = 0.1$, (b) $\mu = 0.01$.

IV. Analytical Model

In this Section, we develop an analytical model of the proposed GMA. We consider a single STA working in saturation and describe its state with the number g_E of groups in state E. We observe g_E at discrete time moments corresponding to the beginnings of beacon intervals. So, the time unit equals one beacon interval.

The algorithm tries to maintain K full groups. Let us consider more general class of GMAs than described in Section II. The GMA arranges reservations according to the vector $\vec{r} = (r_1, r_2, ..., r_K)$, where $r_i > 0$ is the number of reservations in group i. As the STA works in saturation, $\sum_{i=1}^{K} r_i = R$. When at least one reservation from group i is closed, the group becomes blocked, and an empty group is filled with the same number of reservations. In particular, for *uniform distribution*, $r_1 = ... = r_{g'} = \lceil \frac{R}{K} \rceil$ and $r_{g'+1} = ... = r_K = \lfloor \frac{R}{K} \rfloor$, where $g' = R \mod K$. The model allows calculating the amount of advertised information $V(\vec{r}, \mu, K)$ for given $\vec{r}$, μ and K, and hence, for a given μ allows choosing the optimal $\vec{r}$ and K.

Since the lifetime of each reservation has exponential distribution, the probability of closing a particular reservation during a beacon interval equals $1 - e^{-\mu}$ and does not depend on the time during which the reservation already exists. As lifetimes of various reservations are mutually independent random variables, the probability of blocking a group containing r reservations equals the probability of closing at least one of its reservations and can be calculated as follows $p^{block}(r) = 1 - e^{-\mu r}$.

Let $\beta(x) = \{\beta_1, \beta_2, ..., \beta_x\}$ be a subset with cardinality x of the group indices set $\{1, 2, 3, ..., K\}$. The probability of blocking the groups only with indices $i \in \beta(x)$ equals $\tilde{p}^{block}(\beta(x)) = \prod_{i \in \beta(x)} p^{block}(r_i) \prod_{i \notin \beta(x)} (1 - p^{block}(r_i))$. Let $\mathfrak{B}(x)$ be the set of all such subsets. The probability $\hat{p}(x)$ of blocking exactly x groups can be calculated as:

$$\hat{p}(x) = \sum_{\beta(x) \in \mathfrak{B}(x)} \tilde{p}^{block}(\beta(x)). \tag{1}$$

We model the stochastic process $g_E(t)$ with the discrete-time Markov chain. The time moments t and $t+1$ correspond to the beginnings of two consecutive beacon intervals. Let x be the number of groups blocked during the beacon interval $[t, t+1]$ and d be the number of reservations closed during this beacon interval. Below we consider all the transitions from state $g_E(t)$ to state $g_E(t+1)$.

1) If $g_E(t) > 0$, then the following cases are possible.
   a) With probability $\hat{p}(0)$, $x = 0$ and $g_E(t+1) = g_E(t)$.
   b) If $0 < x < g_E(t)$, with probability $\hat{p}(x)$ the STA transits to the state $g_E(t+1) = g_E(t) - x$. Since we consider the STA in saturation, x empty groups are filled instead of the blocked ones.
   c) If $g_E(t) \le x \le K$, $\sum_{i \in \beta(x)} r_i$ reservations are put into $g_E(t)$ free groups somehow. Note that we do not need to know the exact number of reservations in each group after this transition. Actually, when at least one reservation will be closed, SN will be increased and reservations will be divided into groups from scratch as described below in item 2b). So, the STA transits into state $g_E(t+1) = 0$ with probability $\hat{p}(x)$. Therefore, summing over all x values we calculate the transition probability to state $g_E(t+1) = 0$ as $\sum_{x=g_E(t)}^{K} \hat{p}(x)$.
2) If $g_E(t) = 0$, then the following two cases are possible.
   a) With probability $\hat{p}(0) = 1 - p^{block}(R)$, $d = 0$ and the STA remains in state $g_E(t+1) = g_E(t) = 0$.
   b) With probability $p^{block}(R)$, $d > 0$ and the STA transits to the state $g_E(t+1) = G - K$. The SN increases and all the reservations are put into the first K groups.

Using the found transition probabilities, we can find stationary probabilities π_{g_E}.

Now let us find the conditional mathematical expectations $E(V|x, g_E)$ of the amount of advertised information when x groups become blocked in the considered beacon interval (for simplicity, we omit t in the following equations). For $g_E > 0$ and a given $\beta(x)$, the amount of advertised information equals $\sum_{i \in \beta(x)} r_i$, because empty groups are filled instead of the blocked ones with the same number of reservations. So, the corresponding conditional mathematical expectation equals:

$$E(V|x, g_E > 0) = \left\{ \sum_{\beta(x) \in \mathfrak{B}(x)} \tilde{p}^{block}(\beta(x)) \sum_{i \in \beta(x)} r_i \right\} / \hat{p}(x). \tag{2}$$

If in the previous beacon interval $g_E = 0$ and at least one reservation is closed, the SN increases and the STA sends information about all the reservations, so the corresponding mathematical expectation equals:

$$E(V|g_E = 0) = p^{block}(R)R. \tag{3}$$

Finally, the average amount of advertised information can be expressed by the following equation:

$$V = \pi_0 E(V|g_E = 0) + \sum_{g_E=1}^{G-K} \pi_{g_E} \sum_{x=1}^{K} E(V|x, g_E > 0)\hat{p}(x). \tag{4}$$

For a given μ, by varying $\vec{r}$ and K in (4), we can find the optimal parameter values, which minimize V. Note that the exhaustive search of the optimal parameters has high computational complexity. Let us consider a special asymptotic case for which we can provide an explicit formula for the optimal parameter K.

Theorem 1. *For $\mu \to 0$, $\vec{r}$ corresponding to uniform distribution and $R > R^* = \frac{(\sqrt{G}-1)(G+1)\sqrt{(G-1)(G+3)}}{4\sqrt{G}}$, the optimal K which minimizes the amount of advertised information V equals:*

$$K_{opt} = \begin{cases} \frac{G+1}{2}, & for\ odd\ G; \\ \lfloor \frac{G+1}{2} \rfloor\ or\ \lceil \frac{G+1}{2} \rceil, & for\ even\ G. \end{cases} \tag{5}$$

The proof of the Theorem 1 is provided in Appendix.

Despite the rather strict conditions in the theorem formulation, it is of considerable practical importance. Indeed, according to standard [9] $G = 16$ and $R > 83$. For $G = 16$, $R^* = 53$, so the inequality $R > R^*$ holds. Moreover, in Section V, we show that K_{opt} chosen according to (5) also provides a low overhead V for non-asymptotic case as well. So, the GMA can be easily implemented without need for complex parameter adjustment method.

V. Numerical Results

In this Section, we evaluate the performance of the designed enhanced GMA and compare it with the simple GMA proposed in [11]. In all the experiments, we consider the scenario described in Section III. Unless otherwise stated, we set $R = 100$, $G = 16$.

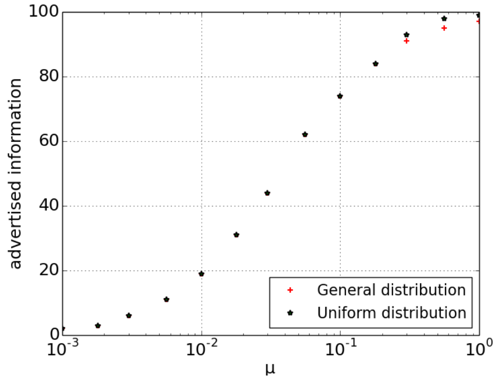

Fig. 3. Comparison of GMA with general and uniform distribution, $R = 100$.

A. Saturated mode

Let us study the case when the STA works in saturation (i.e., when the intensity of incoming data flows $\lambda \to \infty$), using the developed analytical model.

In the first series of experiments (see Fig. 3), we compare the GMA which arranges reservations into groups uniformly with GMAs which arrange reservations according to any other distribution $\vec{r}$ (general distribution). To obtain the red curve ("general distribution"), for a given μ we calculate the minimal amount of advertised information by varying $\vec{r}$ and K. For the black one, we fix $\vec{r}$ (uniform distribution) and choose the minimum over all K values. We can see that the curves are almost coincide (slight difference is observed only for $\mu > 10^{-1}$). So, we recommend the GMA which tries to arrange reservations into K groups uniformly, as described in Section II. In all the following sets of the experiments we consider this GMA.

In the second series of experiments, we compare the GMA which uses a fixed K regardless of μ with the GMA which chooses optimal K by exhaustive search using (4). Fig. 4 shows the relative difference between the amounts of advertised information, provided by the GMA with constant K and by the GMA with optimal K. We can distinguish 3 areas: (i) $\mu \leq \mu^{(1)}_{thresh}$, where $K_{opt} = \left\lfloor \frac{G+1}{2} \right\rfloor = 8$ or $\left\lceil \frac{G+1}{2} \right\rceil = 9$; (ii) $\mu^{(1)}_{thresh} < \mu \leq \mu^{(2)}_{thresh}$, where $K_{opt} = 15$; (iii) $\mu \geq \mu^{(2)}_{thresh}$, where the amount of advertised information is the same for all K values. The analysis for various R (due to the lack of space we do not present these results) shows that thresholds $\mu^{(1)}_{thresh}$ and $\mu^{(2)}_{thresh}$ are inversely proportional to R. It can be seen that for both $R = 100$ and $R = 1000$ the GMA with $K = \{8, 9\}$ provides the minimal overhead in the left area ($\mu \to 0$), which exactly matches the statement of Theorem 1. For other values of μ these K values provide some positive relative difference which does not exceed 10%. So, $K = \{8, 9\}$ can be considered as suboptimal K values regardless of the μ value.

B. Non-saturated mode

Now let us evaluate the performance of the enhanced GMA when STA is not in saturation. We compare GMAs which use optimal K found with different methods: (i) $K = K\_ana$ obtained with the analytical model assuming that $\lambda \to \infty$ (using (4)); (ii) $K = K\_opt$ obtained with simulation by exhaustive search over all possible values; (iii) $K = \{8, 9\}$ obtained from Theorem 1; (iv) $K = 1$ which corresponds to the simple algorithm from [11]. Since the analytical model is developed only for saturated mode, the results presented in this subsection are obtained with simulation. For simulation we use an event-driven custom simulation program, written on C programming language, and consider the scenario described in Section III.

In the experiments, we vary μ over a wide range of values, but, due to the lack of space, in Fig. 5 we present results only for $\mu = \{0.1, 0.01\}$. In particular, when $\mu = 0.01$ the enhanced GMA provides three times lower amount of advertised information than the simple GMA. Choosing $K = K\_ana$ provides the amount of advertised information very close to that provided by $K = K\_opt$. Thus, the assumption that the optimal K chosen in the area where the STA works in saturation provides low amount of advertised information for

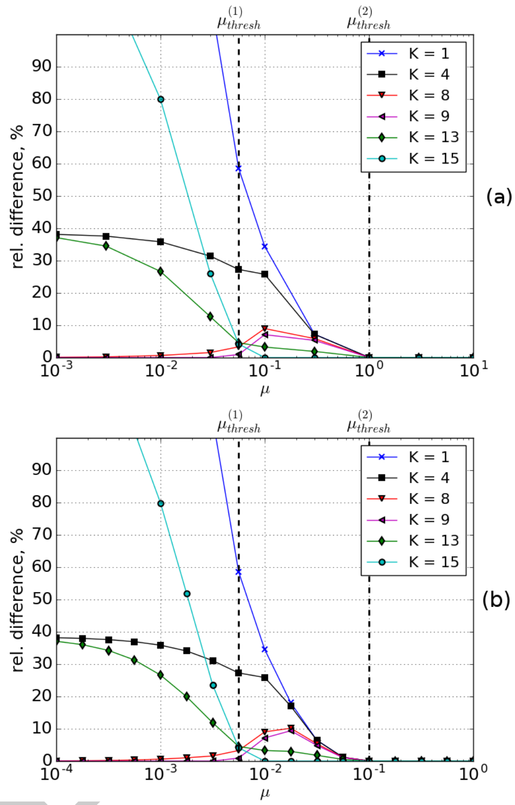

Fig. 4. Comparison of the GMA with fixed K regardless of μ with the GMA which chooses optimal K with analytical model when: (a) $R = 100$, (b) $R = 1000$.

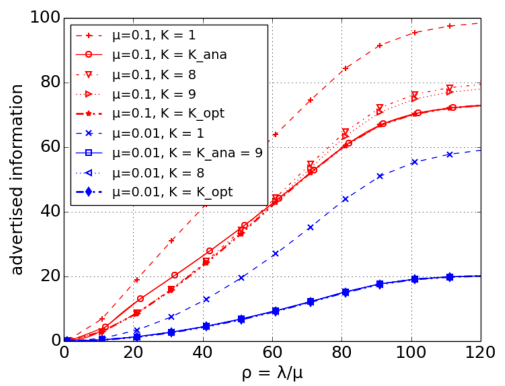

Fig. 5. Comparison of the simple GMA and the enhanced GMA performance when $\mu = \{0.1, 0.01\}$

other values of λ holds. Moreover, the difference between the overhead provided by $K = \{8, 9\}$ and by optimal $K = K_{opt}$ does not exceed 10%. So, finally, we recommend to use the GMA which tries to arrange reservations into $\lceil \frac{G+1}{2} \rceil$ or $\lfloor \frac{G+1}{2} \rfloor$ groups uniformly (the difference in advertised information between these two values is very low). It is important to note, that recommended K depends only on protocol parameter G and can be set regardless of traffic parameters. Hence, the proposed GMA can be easily implemented without any complex method for parameters adjustment.

VI. Conclusion

In this paper, we have studied the problem of reducing the amount of advertised information about channel reservations in Wi-Fi Mesh networks. We have proposed and analyzed the enhanced group management algorithm (GMA), which significantly outperforms the GMA proposed in [11]. We have developed an analytical model for performance evaluation and estimating the optimal parameters values for the proposed GMA. Also, we have obtained an *explicit formula* for the optimal parameter values for an asymptotic case. The results show that the enhanced GMA combined with the analytical method of choosing parameter values allows reducing the amount of advertised information up to 3 times in comparison to the GMA proposed in [11]. Moreover, simulation results show that the enhanced GMA with the optimal parameter (number of full groups) found in the asymptotic case provides close to optimal results for non-asymptotic case. The recommended number of full groups depends only on the protocol parameter G (total number of groups) and can be set regardless of traffic intensity. Hence, the proposed GMA can be easily implemented without need for complex parameters adjustment method. We believe that the approach for sending control information, proposed in IEEE 802.11 standard, which is combined with the developed GMA, will be beneficial not only for MCCA, but for other protocols which require control information dissemination, e.g., routing protocols, channel access protocols, and so on.

Appendix

Proof of Theorem 1: Let us rewrite the transition probabilities of Markov chain in $o(\mu)$ notation. Equation (1) for the probability of closing exactly x groups transforms to:

$$\hat{p}(x) = \begin{cases} 1 - \mu R + o(\mu) & \text{if } x = 0; \\ \mu R + o(\mu) & \text{if } x = 1; \\ o(\mu) & \text{if } x > 1. \end{cases}$$

By neglecting the transitions with probabilities of order $o(\mu)$, we can obtain the following Markov chain. The STA transits from state $g_E(t)$ to state $(g_E(t) - 1) \mod G$ with probability $R\mu$ or remains in state $g_E(t)$ with probability $1 - R\mu$. Since the transition probabilities do not depend on $g_E(t)$, $\pi_{g_E} = \frac{1}{G-K+1}$ for $0 \le g_E \le G - K$.

The distribution $\vec{r}$ corresponds to uniform distribution, i.e., the STA keeps $g' = R \mod K$ groups with $\lceil \frac{R}{K} \rceil$ reservations and $K - g'$ groups with $\lfloor \frac{R}{K} \rfloor$ reservations. Thus, according to (2) and (3), $E(V|g_E = 0) = R \cdot \mu R$ and $E(V|g_E > 0) = \mu(\lceil \frac{R}{K} \rceil)^2 g' + \mu(\lfloor \frac{R}{K} \rfloor)^2 (K - g') = \frac{\mu}{K}(R^2 + g'(K - g'))$. So according to (4) the mean amount of advertised information equals $V(\mu, K, R) = V_1(\mu, K, R) + V_2(\mu, K)$, where $V_1(\mu, K) = \frac{\mu R^2 G}{(G-K+1)K}$, $V_2(\mu, K) = \frac{\mu(G-K)}{K(G-K+1)} g'(K - g')$.

With a given μ and R, function $V_1(\mu, K, R)$ attains the only minimum over the interval $K \in [1, G]$ at $K = \frac{G+1}{2}$. As K can be only integer, to minimize $V_1(\mu, K, R)$ we should choose $K = K_{opt}$ according to (5). Note that function $V_1(\mu, K, R)$ is symmetric with respect to $K = \frac{G+1}{2}$.

Let us calculate the difference $\Delta V_1(\mu, R)$ between the value of function $V_1(\mu, K, R)$ at $K = K_{opt}$ (for even G we consider $K_{opt} = \frac{G}{2}$) and at $K = K_{opt} - 1$. It can be shown that $\Delta V_1(\mu, R) \ge \frac{16\mu R^2 G}{(G-1)(G+1)^2(G+3)}$. Note that for a given μ, if $V_2(\mu, K) < \Delta V_1(\mu, R)$ for all K, then function $V(\mu, K, R)$ attains minimum at the same point ($K = K_{opt}$) as function $V_1(\mu, K, R)$ does. So, we have to prove that for $R > R^*$

$$\max_{K=1..G} V_2(\mu, K) < \Delta V_1(\mu, R). \quad (6)$$

For $V_2(\mu, K)$ we can write the following inequalities: $V_2(\mu, K) < \frac{\mu(G-K)}{K(G-K+1)}(K - 1)K = \mu F(K)$, where $F(Y) = \frac{(G-Y)(Y-1)}{G-Y+1}$. Function $F(Y)$ attains maximum at $Y^* = G + 1 - \sqrt{G}$. So $V_2(\mu, K) \le \mu F(Y^*) = \mu(\sqrt{G} - 1)^2$. Using inequalities for ΔV_1 and for V_2, we obtain that (6) holds when $R > R^* = \frac{(\sqrt{G}-1)(G+1)\sqrt{(G-1)(G+3)}}{4\sqrt{G}}$.

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References

[1] A. Lyakhov, P. Nekrasov, D. Ostrovsky, A. Safonov, E. Khorov, Analysis of the joint use of the proactive and reactive methods of the topology information dissemination in ad-hoc wireless networks, Journal of Communications Technology and Electronics 57 (12) (2012) 1322–1330.

[2] T. Clausen, P. Jacquet, Optimized Link State Routing Protocol (OLSR), RFC 3626.

[3] A. Kiryanov, A. Kureev, A. Lyakhov, E. Khorov, Analysis of logical topology construction mechanisms in manet, Journal of Communications Technology and Electronics 60 (12) (2015) 1379–1388.

[4] F. Borgonovo, A. Capone, M. Cesana, L. Fratta, RR-ALOHA, a Reliable R-ALOHA broadcast channel for ad-hoc inter-vehicle communication networks, Proceedings of Med-Hoc-Net 2002.

[5] C. E. Perkins, P. Bhagwat, Highly dynamic destination-sequenced distance-vector routing (DSDV) for mobile computers, in: ACM SIGCOMM computer communication review, Vol. 24, 1994, pp. 234–244.

[6] R. Ogier, F. Templin, M. Lewis, Topology Dissemination Based on Reverse-Path Forwarding (TBRPF), RFC 3684.

[7] R. Ogier, P. Spagnolo, Mobile Ad Hoc Network (MANET) Extension of OSPF Using Connected Dominating Set (CDS) Flooding, RFC 5614.

[8] Z. Wang, Y. Chen, C. Li, A New Loop-Free Proactive Source Routing Scheme for Opportunistic Data Forwarding in Wireless Networks, IEEE Communications Letters 15 (11) (2011) 1184–1186.

[9] 802.11-2012 - IEEE Standard for Information technology–Telecommunications and information exchange between systems Local and metropolitan area networks–Specific requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications.

[10] A. Ivanov, A. Lyakhov, E. Khorov, Analytical model of batch flow multihop transmission in wireless networks with channel reservations, Automation and Remote Control 76 (7) (2015) 1179–1192.

[11] A. Belogaev, E. Khorov, A. Krasilov, A. Lyakhov, Study of the group-based approach to disseminate control information in wireless networks, in: Proc. IEEE ISWCS'2015.